Massive MIMO Uplink Signal Detector for 5G and Beyond Networks

Robin Chataut
Computer Science Department
Fitchburg State University
Fitchburg, MA, 01420, USA
rchataut@fitchburgstate.edu

Robert Akl
College of Engineering
University of North Texas
Denton, Texas, 76207, USA
Robert.Akl@unt.edu

Utpal Kumar Dey
College of Engineering
University of North Texas
Denton, Texas, 76207, USA
Utpal-KumarDey@my.unt.edu

***Abstract*—Multiple-input Multiple-output (MIMO) technology uses multiple transmitter and receiver antennas for improving throughput and coverage. Massive MIMO is an advancement of MIMO technology that utilizes large numbers of antenna terminals at both transmitter and receiver to provide high spectral efficiency with simple processing. Massive MIMO today has become a captivating technology for future generation networks. Traditional linear, non-linear, and iterative uplink signal detection algorithms for massive MIMO systems do not provide optimal performance, and are computationally inefficient. This paper proposes a fast-convergent detector for massive MIMO systems based on the refinement of the Jacobi iterative method. The proposed algorithm utilizes a novel initialization matrix and a successive over-relaxation (SOR) preconditioner to enhance the convergence of the proposed method. The numerical results verify the optimal error and complexity performance of the proposed algorithm for uplink signal detection in massive MIMO systems.**

***Index Terms*—— 5G, Beyond 5G, 6G, BER, complexity, MIMO, massive MIMO, signal detection**

I. Introduction

MIMO is the foundation of most of the current wireless standards such as Wi-Fi, 4G LTE, LTE-Advanced, and WiMAX [1]- [3]. MIMO systems provide enhanced area throughput compared to the traditional single antenna system. But with the exponential rise in wireless data traffic in the last decade, traditional MIMO has reached its throughput limits. Additionally, with new technologies such as the Internet of Things (IoT), and several smart city applications, there is a colossal increase in wireless data traffic. According to the Ericsson mobility report, there will be more than 370 exabytes of mobile traffic per month [4].

The future generation networks should address the growing demands and provide the user with a high data rate and reliability. Recently, researchers have come up with a novel multiple access technology known as massive MIMO. This technology is considered a core element for future generation networks. Massive MIMO is a state-of-the-art version of the long-established MIMO system, and it uses hundreds of antenna terminals at the base station. With massive MIMO technology, mobile networks can serve several users at the same time with better reliability, coverage, and accuracy. This technology also provides better energy and spectral efficiency, [5]- [8] compared to the traditional MIMO. Massive MIMO uses hundreds of extremely low power antenna terminals; thus, power consumption is not significantly increased with higher antenna providing better energy efficiency [9]- [10]. Massive MIMO also provides beamforming capability,i.e, base station can adapt the antenna radiation pattern. Beamforming helps to focus signals towards the indented users, and with the higher number of antennas, beams become narrower and more concentrated towards the user. This phenomenon helps the user get better quality of service even if the user is at the edge of the hexagonal cell or inside a building. Further, these narrow beams also reduce the interference to the unintended users nearby the designated user. Some other significant benefits that massive MIMO provides are better link reliability, high array gain, low latency, and improved coverage [11]- [12]. Figure 1 shows a massive MIMO system.

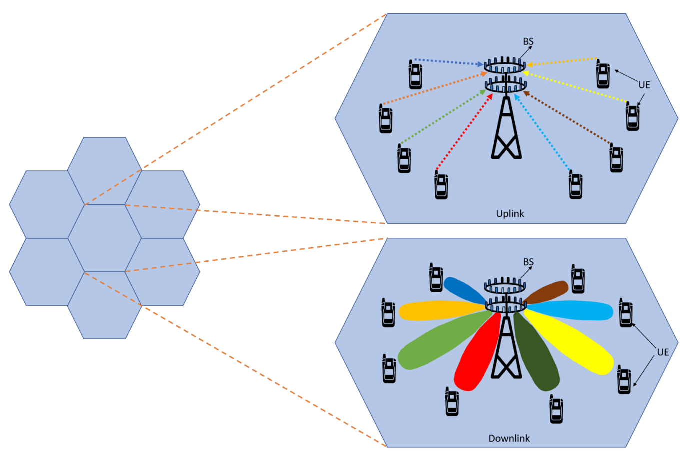

Fig. 1: Massive MIMO.

A. Relevant Prior Art and Motivation

Although we get immense benefits with a massive MIMO system, it comes at the price of increased computational complexity at the base station. The complexity of uplink data detection at the base station increases due to hundreds of antenna terminals at the base station communicating simultaneously with many users. The conventional detectors used for MIMO are not efficient for Massive MIMO systems as their computational complexity rises exponentially with the number of antenna terminals. Recently, there have been several research studies to find computationally efficient and optimal detectors for massive MIMO systems. Linear and non-linear detectors such as Maximum Likelihood (ML), Minimum Mean Square Error (MMSE), Zero-Forcing (ZF), and Successive Interference Cancellation (SIC) provide optimal error performance [13]- [17]. However, these methods involve matrix inversion, and for large antenna systems, matrix inversion is computationally not efficient. Several other iterative and non-iterative algorithms were designed to reduce the complexity. Methods such as Gauss-Seidel (GS) [18], Richardson [20], Neumann Series Approximation (NSA) [21], and Conjugate Gradient (CG) [22] are better in terms of complexity. Some other iterative methods have received significant results [23]-[24]. The conventional Jacobi [19] method has also been used in signal detection, which has the acceptable computational complexity but fails to provide the optimal error performance. Several machine learning and deep learning approaches have been used for complex analysis in massive MIMO. These approaches are mostly in the areas of channel estimation, beamforming, and signal detection. Some of the recent work have received significant results [25]- [29].

Our proposed algorithm is based on the refinement of the conventional Jacobi method. The proposed method provides optimal error performance and complexity. We apply a preconditioner based on the Successive over-relaxation (SOR) method to further accelerate the converge. In addition, we use an initialization vector to further accelerate the algorithm's convergence. The simulation results verify the performance of the proposed algorithm.

B. Paper Outline

The organization of the rest of the paper: Section II-A we describe the system model for the massive MIMO system used for the simulation. Section II-B, we discuss the basics of the Proposed algorithm. In section III, we introduce the steps and parameters used for the simulation. Section III presents the results and analysis and the paper is concluded in section V.

II. System Model and Proposed Algorithm

A. System Model

We have considered a system with N single antenna users communicating. These N users are assumed to be communicating concurrently with a massive MIMO base station having M antennas terminals, where (M≫N). We have assumed perfect channel properties, i.e., perfect Channel State Information (CSI), and to lessen the effect of interference, we have assumed long cyclic prefix [30]. For the channel model, we have assumed the Rayleigh Fading channel model between the users and the bases station. According to the Rayleigh Fading channel model, the signal flowing through the communication channel varies according to the Rayleigh distribution. Each user terminal encodes the information bits. Once the bits are encoded, they are mapped into the constellation point. They are mapped using n-QAM (Quadrature Amplitude) and BPSK (Binary Phase Shift Keying) Modulation [31]. The relationship between the signal transmitted by the users x and the signal received at the receiver (base station) y can be characterized as:

$$y = Hx + n \tag{1}$$

where, $x \in \mathbb{C}^N$ is the signal transmitted by the single antenna user terminals, $y \in \mathbb{C}^M$ is the signal received at the receiver base station, H is the channel vector whose elements $H \in \mathbb{C}^{M\times N}$ are independent and identically distributed (i.i.d) with zero mean and unit variance i.e. $H \sim \mathcal{CN}(0,1)$. Vector n is a circularly symmetric complex Gaussian white noise. Each element of vector $n \in \mathbb{C}^M$ is i.i.d with $n \sim \mathcal{CN}(0,\sigma^2 I)$.The system model is shown in Figure 2.

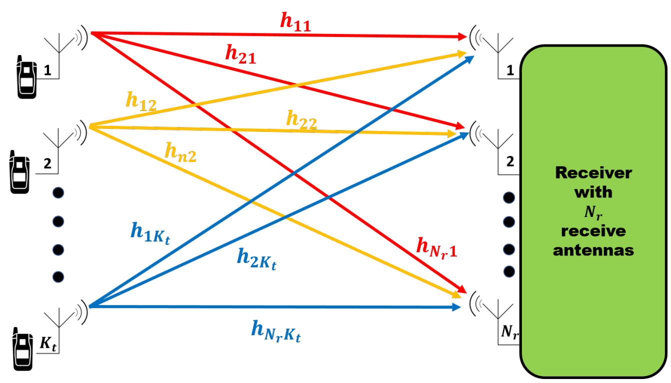

Fig. 2: System model for uplink massive MIMO Systems.

ZF and MMSE are simple linear detectors used for uplink signal detection. ZF eliminates interference but suffers from noise enhancement, whereas MMSE reduces both interference and noise. The ZF detection can be expressed as [39]:

$$x_{ZF} = (H^H H)^{-1} H^H y \tag{2}$$

The MMSE detector can be expressed as [39]:

$$x = (H^H H + \frac{\sigma^2{}_n}{\sigma^2{}_x} I)^{-1} H^H y \tag{3}$$

where, noise variance is represented by $\sigma^2{}_n$ and $\sigma^2{}_x$ is the signal variance. ZF and MMSE both provide optimal error performance but in terms of computational complexity they are not near optimal.

B. Proposed Algorithm

The proposed algorithm is based on the refinement of the Jacobi iterative method. To boost the converge, we introduce a preconditioner based on the SOR method. Additionally, we apply a novel initialization vector to boost the convergence rate. To reduce the complexity, a lot of complex mathematical computations are evaluated outside of the algorithm loop. If we can evaluate complex functions outside the loop, we won't have to do it every iteration. This reduces the computational complexity required for signal processing.

The inputs to the proposed algorithm are y, H, σ, M, N. From (3), the near-optimal minimum mean square detector is :

$$Wx = Y \tag{4}$$

where, $Y = H^H y$ and $W = (H^H H + \frac{\sigma^2{}_n}{\sigma^2{}_x} I)^{-1}$ is called grammian matrix. This is required to avoid the computationally complex matrix inversion in linear methods.

In the Jacobi method, the linear system $Wx = Y$ can be expressed in the form $x = Yx + c$. The coefficient of matrix W can be splitted as as sum of diagonal matrix D, upper triangular matrix U, and lower triangular matrix L:

$$W = D + L + U \tag{5}$$

where . The Jacobi method for the above equation is given as [32]:

$$x_{i+1} = D^{-1}Y - D^{-1}(L + U)x_i \tag{6}$$

where, x_i is the user signal estimated after k_{th} iteration.

The Jacobi method for row strictly diagonally dominant matrix W can converge for any random initial vector x_o.

To improve the converge of the Jacobi method, we do the refinement of Jacobi method. Putting (5) in (6), we get:

$$\begin{aligned}
&Y = Wx \\
&\Rightarrow Y = (D + L + U)x \\
&\Rightarrow Y - Ux = (D + L)x
\end{aligned}$$

Using (5) in above expression, we get:

$$\begin{aligned}
&Y - (W - D - L)x = (D + L)x \\
&\Rightarrow Y(D + L)^{-1} - Wx(D + L)^{-1} + x = x \\
&\Rightarrow x = (D + L)^{-1}(Y - Wx) + x
\end{aligned}$$

Hence, the formula for iterative refinement in the matrix form:

$$x_{i+1} = (D + L)^{-1}(Y - Wx_{i+1}) + x_{i+1} \tag{7}$$

Where, x_{i+1} shown in right hand side of (7) given in (6)

From (6) we can obtain the refinement of Jacobi method in matrix form:

$$x_{i+1} = (I - D^{-1}(L + U))D^{-1}Y + (D^{-1}(L + U))^2 x_i \tag{8}$$

The proof of the (8) is presented in [33], and it converges relatively faster than the traditional Jacobi method and GS method. To further improve the convergence, we apply the SOR preconditioner to the above solution in (8). The improved convergence means fewer iterations required during the loop processing, which helps in reducing the processing time and complexity of the algorithm. The idea is to modify the system $Wx = Y$ into a similar fast converging preconditioned system $\hat{W} h \hat{x} = \hat{Y}$. To apply the preconditioner, we transform the system in (4)

$$C^{-1}Wx = C^{-1}Y \tag{9}$$

where, $C \approx W$ is a non-singular matrix. The value of preconditoner C should be chosen such that $\hat{W} = C^-1W$ [34]. In this paper, we have used SOR preconditioner:

$$C_{SOR} = \frac{1}{p}(D + pL) \tag{10}$$

where p is the relaxation parameter which can be approximated as [35]:

$$p = \frac{2}{1 + \sqrt{2\left(1 - \left(\left(1 + \sqrt{\frac{N}{M}}\right)^2 - 1\right)\right)}} \tag{11}$$

Here, convergence proof of (11) is presented in [35]. The value of the relaxation parameter is based on the value of M and N. Since the value of M and N is initially fixed when the MIMO configuration is set, its value doesn't change when the channel state is changed. The relaxation parameter is initially computed once outside the algorithm iteration when the MIMO configuration is set. Thus relaxation parameter has a minimal effect on the computational complexity of the algorithm.

In general, iterative algorithms use zero vectors to initialize the final estimation. But, initializing the system with zero vector required more iteration to come near the final solution, thus degrading the algorithm's convergence. If the initial estimate is closer to the final solution, we can significantly reduce the number of iterations required, improving the overall algorithm's convergence. The initial solution we are using is based on the values that we have computed in earlier steps of the algorithms, i.e., D, L, and U:

$$x_0 = \frac{Y}{(D - L - U)} \tag{12}$$

Since the values of a L, U, and D matrix are computed in previous algorithm steps, it adds minimal computational cost and improves the convergence of the algorithm. Algorithm 1 summarizes the proposed algorithm.

III. Simulation Setup

For conducting the simulations, we have assumed a massive MIMO base station having 16 to 512 antenna terminals. The base station antenna terminals are supposed to be communicating concurrently with 16 users having a single antenna. We compared the error performance (BER) and complexity of the proposed algorithm with convectional iterative algorithms like MRC, Jacobi, CG, and GS. ML algorithm, which provides the theoretical maximum error performance, is used as a benchmark algorithm. Table I lists the parameters used for the simulations. We have used a system bandwidth of 20 MHz, and the frame duration of 10ms and slot duration of 0.5 ms is used.The uncorrelated Rayleigh fading is used as

Algorithm 1 Proposed Algorithm

Inputs: $y, H, {\sigma^2}_n, {\sigma^2}_x, N, M$

Pre-processing Steps:

1) Compute the grammian matrix: $W = (H^H H + \frac{{\sigma^2}_n}{{\sigma^2}_x} I)$
2) Evaluate $Y = H^H * y$
3) Make matrix W sparse: $\hat{W} = sparse(W)$
4) Evaluate the D, U and L matrices:
   $D = diagonal(\hat{W})$
   $U = upper(\hat{W})$
   $L = lower(\hat{W})$
5) Evaluate relaxation parameter:
   $p = \frac{2}{1+\sqrt{2\left(1-\left(\left(1+\sqrt{\frac{N}{M}}\right)^2-1\right)\right)}}$
6) Evaluate SOR preconditioner: $C_{SOR} = \frac{1}{p}(D + pL)$
7) Apply preconditioner:
   $\tilde{W} = C_{SOR}^{-1} * \left(\hat{W}\right)$
   $\tilde{Y} = C_{SOR}^{-1} * (Y)$
8) Compute $\tilde{D}$, $\tilde{U}$, and $\tilde{L}$ from $\tilde{W}$
   $\tilde{D} = diagonal(\tilde{W})$
   $\tilde{L} = lower(\tilde{W})$
   $\tilde{U} = upper(\tilde{W})$
9) Estimate the initial solution: $x_0 = \frac{Y}{(D-U-L)}$

Algorithm iteration:

10) **for** $i = 1$ to i_{max} **do**
11) $x_{i+1} = (I - \tilde{D}^{-1}(\tilde{L} + \tilde{U}))\tilde{D}^{-1}\tilde{Y} + (\tilde{D}^{-1}(\tilde{L} + \tilde{U}))^2 x_i$
12) **End for**

Estimated output: x_i

the channel model with carrier frequency of 2.5 GHz. Four different modulation schemes have been used to conduct the simulations: BPSK, QPSK, 16QAM, and 64QAM. A signal variance of 2.5 is used, while noise variance is dependent on the value of SNR (Signal-to-Noise Ratio). The SNR of 0f 0dB-25dB is used for the simulations. Some general simulation steps are shown in Figure. 3.

TABLE I: Simulation Parameters.

Parameter	Value
Base Station Antenna Terminals	16 to 512
Mobile Users	16
Bandwidth	20 MHz
Frame duration	10 ms
Slot duration	0.5 ms
Carrier Frequency	2.5 GHz
Variance in User Signal	2.5
SNR	0 dB - 25dB
Modulation Scheme	BPSK, QPSK, 16QAM, 64QAM
Channel Model	Uncorrelated Rayleigh Fading

IV. Simulation Results

A. BER Performance

BER is widely used to measure the performance of iterative algorithms. BER is computed as the ratio of the number of error bits to the number of total bits [36]. The BER performance with 32 base station antenna is shown in Figure. 4. The simulation was conducted with 16 users and 16QAM modulation. The performance of the proposed algorithm is near-optimal (ML), and it is better than traditional signal detection algorithms. For example, at BER = 10^-2, a gain of 4.3dB was achieved by the proposed algorithm compared to the GS algorithm. Furthermore, with 64 antenna terminals at the base station, the error performance of most of the algorithms got better, as shown in Figure. 5. There was no significant change in MRC and Jacobi algorithms because the number of antennas has little to no effect on the performance of these algorithms. This improvement on algorithms other than MRC and Jacobi is due to the higher array gain with more base station antenna terminals. As a result, the performance of our proposed algorithm is closer to the optimal. For example, if we look at BER = 10^-2, the proposed algorithm has a more-or-less, better-improved performance over the GS method, and it is outperforming the CG method by gaining a 4dB SNR.

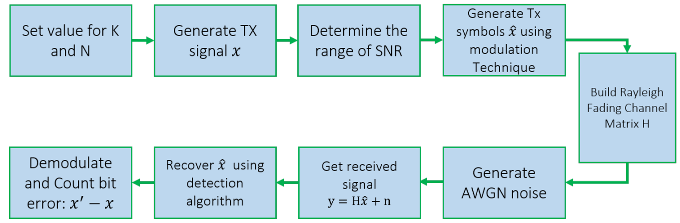

Fig. 3: Summary of simulation steps.

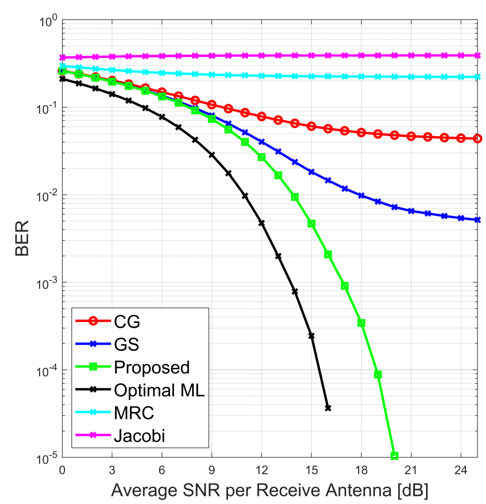

Fig. 4: BER vs. Average SNR performance with 32 base station antennas.

For our next experiment, we gradually increased the number of antenna terminals at the massive MIMO base station. The BER performance with the increasing number of base station antenna terminals is shown in Figure. 6. For this experiment, we have used 16 QAM modulation and 16 single-antenna

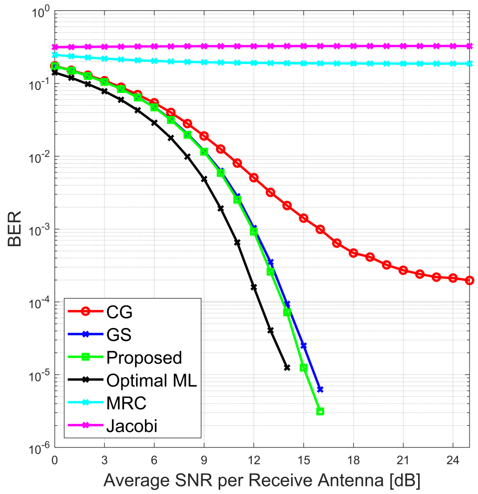

Fig. 5: BER vs. Average SNR performance with 64 base station antennas.

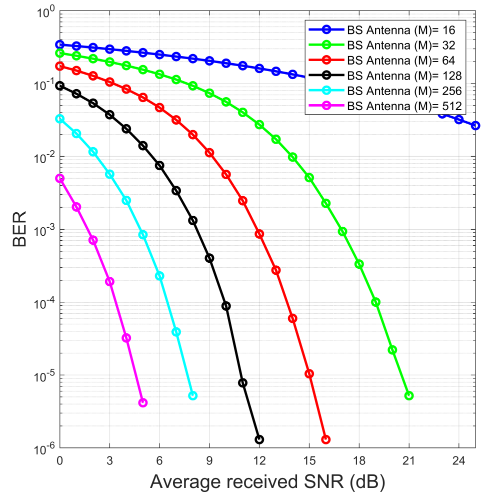

Fig. 6: BER vs. SNR performance with different base station antenna terminals.

users. The BER performance has improved considerably for all the algorithms due to the higher array gain we get with more antennas. For example, in Figure. 6, at BER = 10^-3, the 512 antennas system has 10.2 dB gain compared to 64 antennas system and 15.1 dB gain compared to 32 antenna system. But with the increase in antenna terminals, the computation complexity also increases at the base station. More on complexity is discussed in Section IV-B.

Finally, we conducted simulations with different modulation schemes. For this experiment, we have utilized a 32 antenna massive MIMO system along with 16 users. Figure. 7 shows the performance with various modulation schemes in this configuration. The simulations results show that the BER performance is best with BPSK modulation, whereas it is worst with 64-QAM modulation. However, the higher modulation order provides a higher data rate which may be helpful for several real-time applications. Thus, choosing an optimal modulation order is an important design consideration and should be decided based on the real-world application.

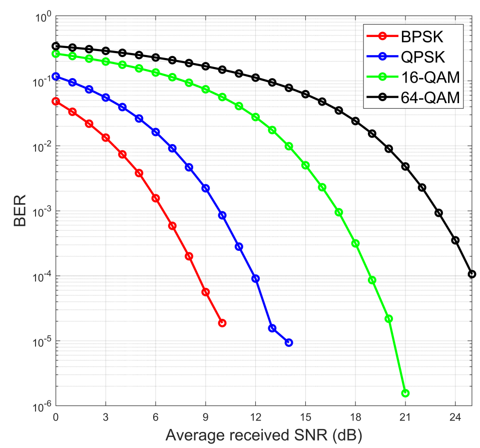

Fig. 7: BER VS. SNR performance with different modulation schemes.

B. Complexity Performance

To assess the computational complexity of the algorithms, we have used Big $\mathcal{O}$ time complexity notation. Big $\mathcal{O}$ usually represents the worst-case scenario of the algorithm [37], [38], [40], [41]. This notation gives us an idea of the execution time taken by the algorithm based on the number of steps required to complete it and the size of the input. Our proposed algorithm is iterative; its complexity mostly depends on the number of loop iterations during the signal processing. If i is the number of loop iterations, the Big $\mathcal{O}$ of the proposed algorithm was evaluated in the order of $\mathcal{O}N^2i$. The complexity of CG, GS, and Jacobi algorithms are also similar since the proposed algorithm needs a few iterations ((~ 3) to get to the final solution. However, the traditional linear algorithm like ZF, MMSE, and ML have very high computational complexity in the order of $\mathcal{O}(MN^2)$ [42], mainly depending on the number of the base station antennas. As the number of base station antenna terminals ($M \geq 16$) is much higher than the number of iterations ((~ 3), the complexity of the proposed algorithm is found to be better than the conventional iterative algorithms.

TABLE II: Computational Complexity.

Algorithm	Complexity
MMSE, ZF, ML	$\mathcal{O}(MN^2)$
CG	$\mathcal{O}(MN^2)$
GS, Jacobi	$\mathcal{O}(N^2i)$
Proposed	$\mathcal{O}N^2)i$

V. Conclusions

This paper proposes an algorithm for user signal detection using the refinement of the Jacobi method. The initial acceleration was achieved using a preconditioner based on the SOR method, whereas an initialization vector was used to further speed up the convergence. We analyzed the performance of the proposed algorithm in various scenarios by comparing it with several existing iterative and non-iterative algorithms like CG, GS, MRC, Jacobi, and ML. The numerical results from the simulation showed that the proposed algorithm has improved performance over conventional algorithms. The results from the simulation showed that the performance improves larger base station antennas, whereas the low modulation order performed better in terms of error performance. However, it should be noted that the higher modulation order provides a higher data rate which may be helpful for several real-time applications. Thus, choosing an optimal modulation order is an important design consideration and should be decided based on the user application. These novel signal detection algorithms will help us realize the goals of 5G, 6G, and beyond networks.

Our future goal is to test this simulation with many multi-antenna users. We also want to test this simulation with more base station antennas and introduce more real-time network parameters. There are still many challenges and questions before we realize the full potential of massive MIMO systems. Indeed unintended outcomes and novel challenges abound.

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